Equity and Bond Comovements: A Machine Learning Perspective

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Abstract

We study the comovements between stocks and bonds by focusing on Treasury bonds and corporate bonds separately. The stock-Treasury bond correlation transitions from positive to negative while the correlation between stocks and high-yield corporate bonds consistently remains positive displaying a notable increasing pattern. Employing machine learning techniques, we find that inflation and bond illiquidity contribute the most to the positive stock-Treasury correlation while the negative scenario is largely explained by the cross-market hedging phenomenon. Default risk and bond illiquidity emerge as crucial characteristics influencing the correlation between stocks and high-yield corporate bond returns. Utilizing machine learning approaches and an extensive panel of characteristics, we provide a comprehensive and objective assessment on the determinants of stock-bond correlation.

JEL Classification: G12, G17, P16, E44

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1 Introduction

As of the year 2022, the total market capitalization of equity markets in the United States stood at approximately \$40.20 trillion, compared to the \$25.00 trillion worth of Treasury bonds and the \$10.6 trillion value of corporate bonds (SIFMA, 2023). The three substantial figures represent a great portion of the world's total wealth. The correlations among these three markets represent pivotal elements in the decision-making process of asset allocation, significantly influencing various aspects, ranging from the diversification of risks across multiple asset classes to the determination of expected premia associated with these risks. Particularly noteworthy is the diminishing negative correlation between stocks and Treasury bonds in light of the recent inflationary surge, thereby provoking apprehensions regarding the potential onset of a regime shift, which might undermine the diversification characteristics inherent in a multi-asset portfolio. Consequently, comprehending the comovements between stocks and bonds is of paramount interest to both the academic community and practitioners alike.

Recent studies (Campbell, Pflueger, and Viceira, 2020; Li, Zha, Zhang, and Zhou, 2022; Duffee, 2023) document that the correlation between market portfolio of stock and long-term Treasury bond in the United States has switched from positive to negative since the end of the 20th century (Panel A of Figure 1). Meanwhile, we find the market portfolio's correlation between stock and high-yield (henceforth, HY) corporate bond evolve increasingly positive (Panel B of Figure 1) from around 0.10 in the early 2000s to around 0.50 in the early 2020s.

These studies are particularly intrigued by the question of why the time-varying correlation between stocks and Treasury bonds switch sign after the late 1990s. Several explanations have been explored in this context. Campbell, Pflueger, and Viceira (2020) attribute this shift to the correlation between inflation and the output gap, with a heightened sensitivity of monetary policy to changes in the output gap. While risk premium amplify this regime switch in stockbond comovement. Li, Zha, Zhang, and Zhou (2022) propose a model that incorporates varying activeness in both monetary and fiscal policies, highlighting technological shocks and investment stocks as the primary driving forces behind the change. In an earlier study conducted by Baele, Bekaert, and Inghelbrecht (2010), they emphasize the significance of liquidity in both stock and Treasury markets as key factors influencing the stock-Treasury correlation.

Considering the intricate nature of the macroeconomic system, the existing research has yielded valuable insights into unraveling the enigmatic shifts in stock-Treasury correlations. Nonetheless, in the presence of a multitude of variables within the macroeconomic framework and the coexistence of various explanatory factors, incorporating all factors in a theoretical model is challenging, it remains an empirical question to discern which variables exert the most significant influence on determining the stock-Treasury correlation. Furthermore, existing studies on stock-bond correlation has primarily concentrated on elucidating the dynamics of the stock-Treasury correlation. Remarkably, there has been a dearth of attention directed towards exploring the correlation between stocks and corporate bonds, especially stock-HY correlation. In this work, we aim to provide empirical analysis to these questions: Why the comovements of stock and Treasury behave differently, while stock and HY bond behave more and more similarly? How do the driving forces evolve? Do these two empirical facts share any common economic underpinnings?

The challenges to address above question arise from the nature of correlation estimation. To attain a precise estimation of correlation, econometricians require a relatively long-sample periods for bivariate time series. Previous studies, such as Baele, Bekaert, and Inghelbrecht (2010), Colacito, Engle, and Ghysels (2011) usually employ the daily returns to generate quarterly correlations. Even with data spanning back to the 1960s, scholars could only obtain around 200 time-series observations, in this study the number is 208 (Table 2). It is feasible to utilize dynamic factor model methods, as employed Bekaert, Engstrom, and Grenadier (2010) and Colacito, Engle, and Ghysels (2011), when analyzing a limited number of variables. A macroeconomic general equilibrium framework becomes the preferred choice when establishing a clear theoretical foundation for discussing the economic channels. Nevertheless, these approaches may not be sufficient when dealing with a broad panel of variables that could influence stock-bond

correlations. Recent progress in the application of machine learning method in financial economic analysis provides solutions to these challenges. Leveraging machine learning techniques, including dimension reduction, regularization, and penalization methods as in Giglio and Xiu (2021), Huang, Li, and Wang (2021), Kelly, Pruitt, and Su (2019), enables us to conduct robust empirical analyses on a substantial number of time-series variables. The empirical exercises are as follows.

We first model the quarterly realized and MIDAS correlation between stock, ten-year Treasury bond and HY corporate bond excess returns based on the Colacito, Engle, and Ghysels (2011) (Dynamic Conditional Correlation) DCC- (Mixed-Data Sampling) MIDAS model. The endogenous smoothing technique allows us extract the long-run component from high-frequency daily stock and bond returns, filtering out the short-run noise and enhancing the prediction of machine learning model. We collect an extensive panel of characteristics that are documented influencing stock-bond correlations, including inflation (David and Veronesi, 2013; Song, 2017), macroeconomic correlation (Campbell, Pflueger, and Viceira, 2020; Li, Zha, Zhang, and Zhou, 2022), volatility (Bansal, Kiku, Shaliastovich, and Yaron, 2014), uncertainty(Connolly, Stivers, and Sun, 2005; Bekaert, Engstrom, and Xing, 2009), illiquidity (Baele, Bekaert, and Inghelbrecht, 2010) and firm leverage(Huang and Huang, 2012; Nietoa and Rodriguez, 2015). To identify the importance of each individual candidate characteristic, we employ five advanced machine learning techniques, including Least Absolute Shrinkage and Selection Operator (LASSO), Ridge Regression (Ridge), Elastic Net Regularization (ENet), Principal Component Regression (PCR), and Partial Least Square Regression (PLS). All these approaches have been shown efficient in variable selection and dimension reduction (Gu, Kelly, and Xiu, 2020; Giglio and Xiu, 2021).

In the context of stock-Treasury correlation, our analysis reveals distinct patterns in two different scenarios. In the positive scenario (1969 Q1–1997 Q4), three inflation-related measures stand out as top-ranking characteristics, coupled with the bond illiquidity. Our finding demonstrate that it is the Treasury bond market fluctuations that primarily drive the stock-Treasury comovement. While in the negative scenario (1998 Q1–2020 Q4), the most influential characteristics all relate to the phenomenon of cross-market hedging. Increased stock volatility, higher firm leverage, stock illiquidity and elevated uncertainty collectively motivate investors to migrate from risky assets (stocks) to safer options (Treasury bonds). Treasury bonds emerge as a safe haven against elevated levels of risk and uncertainty, resulting in a negative return correlation. We also employ rolling window analysis to further corroborate the robustness of our results.

Given the starkly different upward trend observed in the stock-HY correlation in contrast to the stock-Treasury correlation, we repeat the machine learning analysis using the stock-HY correlation as the target. The results indicate the significant role of firm leverage in determining the comovement between stock and HY corporate bond, both operating leverage and market leverage consistently emerge as high-ranking characteristics across all five machine learning methods. Furthermore, macroeconomic uncertainty and bond illiquidity also play a vital role in influencing the stock-HY correlation. As stocks and HY corporate bonds are both claims on firms' assets, investors pay first-order attention on the default risk and firm's fundamentals when holding these risky securities. Consequently, any change in firms' financial health may lead the returns of stocks and corporate bonds to move in tandem, resulting the positive correlation observed between stocks and HY corporate bonds.

We continue to investigate the economic contributions using LASSO approach. For stock-Treasury correlation, we find inflation is the most impactful contributor with the model fit deteriorates by more than 80% when omitting the inflation measures. Volatility and illiquidity measures contribute the most among non-macroeconomic variables. Regarding stock-HY correlation, nonmacroeconomic variables contribute much more than macroeconomic variables, especially firm characteristics (e.g., leverage, idiosyncratic volatility). These findings reaffirm our previous findings from a quantitative perspective. To delve deeper, we proceed to perform LASSO model selection tests on five top-ranking characteristics, decomposing correlation into stock volatility, bond volatility, and covariance as the target. For stock-Treasury correlation, the results indicate that most of the characteristics significantly affect bond volatility in pre-break period, but they exert a greater influence on the covariance component in the post-period break. Bond market and the cross-market hedging driven the stock-Treasury correlation variation during two periods respectively. For stock-HY correlation, the top characteristics load on the covariance component, implying commonality between stock and HY corporate bond.

This study contributes to several strands of literature. First, it adds to the body of research examining the stock-bond correlation. Numerous studies explore the stock-Treasury correlation and its underlying determinants while fail to reach a consensus. Earlier research, such as Connolly, Stivers, and Sun (2005), highlights that stock market uncertainty is a major contributor to the negative stock-Treasury correlation while Baele, Bekaert, and Inghelbrecht (2010) demonstrates the crucial role of liquidity proxies, with macroeconomic fundamentals playing a less significant role. Recently, Campbell, Pflueger, and Viceira (2020) establishes a theoretical framework linking the correlation to inflation and the output gap. Li, Zha, Zhang, and Zhou (2022) analyses the impacts of different levels of monetary and fiscal policy, identifying technological shocks and investment stocks as primary drivers of both positive and negative scenarios. Using a productionbased equilibrium model, Kozak (2022) finds that "flight-to-safety" effect explains the negative stock-Treasury correlation while technology diversification contributes to the positive correlation. While almost all the previous studies start from the theoretical aspect, we initiate from a distinct angel that let the data reveal the truth. We employ cutting-edge machine learning techniques on all the documented determinants in the literature to comprehensively assess their contribution in explaining the stock-Treasury correlation. Consequently, our research provides a more cohesive and objective resolution to the ongoing debate and arguments in the current literature. Furthermore, while the majority of current research focuses on the comovement of stock and Treasury bond returns, very few studies examine the relationship between stock and corporate bond returns. As one exception, Dickerson, Fournier, Jeanneret, and Mueller (2022) explains the stock-corporate bond correlation from the perspective of default risk. Our study fills in this void by exploring the determinants of correlation between stocks and HY corporate bonds and providing novel insights

that inspire future research in this area.

Second, this study connects to the application of machine learning methods in economic and finance. Machine learning techniques have gained widespread application in financial research with a focus on return predictability. For instance, Huang, Li, and Wang (2021) develop an aggregate disagreement index using partial least square method and empirically demonstrate its predictive power on market returns. Gu, Kelly, and Xiu (2020) and Giglio and Xiu (2021) conduct of machine learning techniques for measuring risk premiums and showed their superiority in reducing measurement error and enhancing forecasting accuracy. While existing literature predominantly focus on relatively high-frequency asset returns, very few studies have applied machine learning methods to macroeconomic variables. Macroeconomic variables typically involve low-frequency data, making it challenging to perform ordinary least squares (OLS) regressions on a large panel of such variables due to the potential overfitting issue. Therefore, the application of machine learning techniques to macroeconomic variables is well-suited, given that these methods are designed for variable selection and dimension reduction. Consequently, our study introduces a novel approach by employing machine learning methods in a macroeconomic context, which advances their application in the field and enhances our understanding of stockbond correlation.

This paper is organized as follows. Section 2 introduces the data and methodology. Section 3 shows that the correlation between stocks and Treasury bonds switches from positive to negative while correlation between stocks and HY corporate bonds consistently remains positive during the sample period. Section 4 analyses the importance of multiple characteristics in explaining the stock-Treasury and stock-HY correlation. Section 5 concludes.

2 Data and Methodology

2.1 Data and Sample

This section describes the data and sample in our analysis, including the stock-Treasury correlation, Treasury-HY correlation and thirty economic characteristics for the United States. Our main sample period is from 1969 Q1 to 2020 Q4, for a total of 208 observations. In the analysis involving HY corporate bond, the sample period spans from 1994 Q2 to 2020 Q4.

Stock-Bond Correlation We obtain daily NYSE/AMEX/NASDAQ value-weighted returns including dividends from CRSP. The stock excess returns are in excess of the U.S. three-month Treasury bill rate as the proxy for the risk-free rate. Regarding bond excess returns, we primarily include U.S. government Treasury bond and HY corporate bond. We exclude investment-grade corporate bond (henceforth IG), mainly due to its comovement with stock being similar to that of Treasury bond, whereas HY corporate bonds exhibit a distinctly different pattern. The Treasury excess returns are the ten-year Treasury bonds rate from CRSP, also in excess of U.S. three-month Treasury bill rate. We choose longer-term bonds over shorter-term ones because long-term bonds are closer maturity substitutes to stocks and monetary policy operations are more likely to have a confounding influence on shorter-term bonds (Connolly, Stivers, and Sun, 2005). The HY bond excess returns are ICE BofA US high yield index return, minus the risk-free rate.

Next, we compute the quarterly realized correlation and MIDAS correlation between stock, Treasury and HY bond excess returns based on the DCC-MIDAS model (Colacito, Engle, and Ghysels, 2011). This model combines of the Engle (2002) DCC model which incorporates dynamic conditional correlation, with the Engle, Ghysels, and Sohn (2013) GARCH-MIDAS model, featuring mixed data sampling of variance. In essence, The DCC-MIDAS model enables us to extract long-run correlation component from daily stock and bond returns, capturing the fundamental or underlying causes of time variation in correlation. One benefit of this approach compared to a pure smoothing method like wavelets is that in DCC-MIDAS model, the optimal degree of smoothing is endogenously determined. Following Colacito, Engle, and Ghysels (2011), we estimate the long-run stock-bond correlation in two steps. In the first step, we estimate conditional stock and bond return variances using univariate GARCH-MIDAS models separately. In the second step, returns are standardized by subtracting the estimated means and dividing by the conditional variances to obtain the standardized residuals. We can calculate condition stock-bond correlations based on these standardized residuals. Appendix A details the technical details.

Macroeconomic Characteristics We construct a bunch of macroeconomic characteristics covering various aspects including inflation, economic growth, interest rates, macroeconomic correlation and the output gap. The detailed definition of characteristics is given in Table 1 and the summary statistics are shown in Table 2. In standard asset pricing models, the fundamental drivers of stocks and bonds returns can typically be categorized into two aspects: those that affect cash flows and those that affect discount rates. Inflation can simultaneously affect both stock and bond returns. Bonds, in particular, are directly affected by inflation. Since bonds provide fixed nominal cash flows, the value of these cash flows decreases as inflation rises, leading to higher bond yields and lower bond prices as compensation. The impact of inflation on stocks is more nuanced. On one hand, the nominal income from dividend payments increases with rising inflation. However, higher inflation also implies a higher discount rate due to inflation illusion (Modigliani and Cohn, 1979; Campbell and Vuolteenaho, 2004). Generally, many empirical studies find a negative relation between inflation and real stock returns (Bodie, 1976; Fama and Schwert, 1977; Gultekin, 1983).

The growth measures consist of the growth of cash dividends, industrial production, non-farm payroll, corporate profits and the unemployment rates. These characteristics are closely associated with stock returns as positive growth shocks lead to increased future income and rising stock prices. For bond returns, they are highly sensitive to interest rates and monetary policy. In our analysis, we incorporate the real federal funds rate, as well as the monetary policy gap between federal funds rate and the neutral rate of interest or the optimal rate implied by the Taylor rule as measures of interest rates. In the theoretical framework proposed by Campbell, Pflueger, and Viceira (2020),

the shift in correlation between inflation and output gap contributes to the comovement of stock and bond returns. Therefore, we calculate not only the output gap but also the correlation between inflation and three macroeconomic variables: output, output gap, and industrial production to capture the dynamics among theses fundamental factors.

Risk-related Characteristics In addition to the standard macroeconomic variables, our analysis incorporates various risk-related characteristics to provide a comprehensive understanding of stock-bond return correlations. These risk-related characteristics fall into five categories: volatility, uncertainty, risk aversion, and the variance premium, as well as illiquidity measures for both the stock and bond markets. Bansal, Kiku, Shaliastovich, and Yaron (2014) find that an increase in macroeconomic volatility is associated with an increase discount rate, playing a significant role in explaining return dynamics. We construct volatility using actual data for the S&P 500 index and risk-free interest rates.

Uncertainty may affect risk premiums and equity valuation, though the direction of influence is ambiguous (Bekaert, Engstrom, and Xing, 2009). We construct the uncertainty measures using data from the Survey of Professional Forecasters (SPF), which includes forecasts dispersion for output, inflation, industrial production, and unemployment rates. Additionally, we include macroeconomic uncertainty measure from Jurado, Ludvigson, and Ng (2015). Risk aversion and variance premium are closely related concepts, both of which are crucial in predicting stock and bond returns. The risk aversion we utilize is from Bekaert and Engstrom (2010) and Bekaert, Engstrom, and Xu (2022), who have constructed an empirical proxy for risk aversion coefficient based on habit-like models. The variance premium are calculated by subtracting the fitted MIDAS variance from the VIX squared following Baele, Bekaert, and Inghelbrecht (2010).

Illiquidity is considered one of the dominant factors influencing stock-bond correlation and covariance dynamics (Baele, Bekaert, and Inghelbrecht, 2010). There are three potential channels. The first involves altering the speed at which economic shocks transmit through the market; in illiquid markets, returns may struggle to respond rapidly to economic shocks. The second channel

involves market pricing of liquidity, where positive liquidity shocks enhance market returns. The third channel is the "flight-to-safety" effect; during crises, investors may shift from illiquid assets to highly liquid ones, resulting in negative stock-bond correlations. We use the "zero return" measure for stock illiquidity, which is the capitalization-based proportion of firms with zero daily returns across the market in a month, taken at the end of each quarter (Lesmond, Ogden, and Trzcinka, 1999). For bond illiquidity, we calculate the monthly average daily quoted spreads across securities with multiple maturities, taken at the end of each quarter (Goyenko and Ukhov, 2009).

Firm-level Characteristics Treasury bonds and HY corporate bonds have entirely different underlying risk factors, with the former being interest rate risk and the latter being credit risk associated with potential defaults. Credit risk accounts for only a small fraction of Treasury-IG bond yield spreads, whereas it accounts for a much higher fraction of Treasury-HY bond yield spread (Huang and Huang, 2012). Therefore, we incorporate more firm-level characteristics to capture the credit risk of HY corporate bonds: firm leverage, idiosyncratic risk, credit spread and investor sentiment. The leverage ratio is a key consideration to lenders and investors: high leverage is associated with higher financial instability and increased credit risk. We differentiate two types of leverage ratio here. Market leverage is total liabilities divided by market value of equity, value weighted across Compustat firms. Operation leverage is value weighted sum of administrative expenses and cost of goods sold, scaled by total assets (Novy-Marx and Notes, 2011). Evidence on individual stock and bond correlation has find the correlations are positively associated with firm risk measures, especially idiosyncratic stock risk and financial leverage (Nietoa and Rodriguez, 2015). Since we focus on the stock-bond correlation in the market index level, we constructed value-weighted firm idiosyncratic risk across all Compustat firms, as well as the credit spread between BAA and AAA-rated corporate bond yields as a supplementary risk measure (Goyal and Welch, 2008). Bethke, Gehde-Trapp, and Kempf (2017) document sentiment deterioration leads to higher correlation between bond and risk factors, which translates into increasing IG-HY bond correlation. The investor sentiment index we use is taken from Baker and Wurgler (2006).

[Insert Table 1 and 2 about here]

2.2 Methodology

Following Gu, Kelly, and Xiu (2020), we compare and evaluate a variety of machine learning methods, including the penalized linear regression methods such as least absolute shrinkage and selection operator (LASSO), ridge regression (Ridge), and elastic net (ENet); dimension reduction techniques such as principal component analysis (PCA) and partial least square (PLS). Below, we briefly describe the five machine learning methods.

Assume that the correlation of stock and bond returns can be explained by multiple characteristics:

$$\operatorname{Corr}_{t} = g^{*}(z_{t}) + \varepsilon_{t} \tag{1}$$

where g(.) is a flexible function of *P* characteristics, i.e, $z_t = (z_{1,t}, ..., z_{P,t})'$. Under the most commonly-used linear regression method, $g^*(.)$ can be approximated by a linear function as:

$$g(z_t; \theta) = z'_t \theta, \tag{2}$$

where $\theta = (\theta_1, ..., \theta_P)'$ can be estimated by the ordinary least squares (OLS) via the following optimization problem:

$$\min_{\theta} L(\theta) \equiv \frac{1}{2T} \sum_{t=1}^{T} (\operatorname{Corr}_{t} - g(z_{t}; \theta))^{2}$$
(3)

The estimate of θ is unbiased and efficient if the number of characteristics is relatively small, while *T* is relatively large. However, existing literature documents a large number of characteristics that affect stock-bond correlation. This suggests that using the traditional OLS regression in our context will lead to overfitting issue and the estimates are inefficient and even inconsistent. Hence, we use five machine learning methods that have been recently used in the finance literature.

2.2.1 Penalized Linear Regression: LASSO, Ridge and Elastic Net

To address the overfitting issue, a commonly used method involves adding a penalty term to the objective function in Eq. (3). This penalty term helps estimate θ by minimizing the following modified objective function:

$$\min_{\boldsymbol{\theta}} L(\boldsymbol{\theta};,.) \equiv L(\boldsymbol{\theta}) + \boldsymbol{\varphi}(\boldsymbol{\theta};.), \tag{4}$$

Here, $\varphi(\theta;.)$ represents the penalty applied to θ . The functional form of $\varphi(\theta;.)$ determines the extent to which certain elements of θ are regularized and shrunk towards zero. A general penalty function in machine learning is:

$$\varphi(\theta;\lambda,\rho) = \lambda(1-\rho)\sum_{j=1}^{P}|\theta_j| + \frac{1}{2}\lambda\rho\sum_{j=1}^{P}\theta_j^2,$$
(5)

where $\lambda > 0$ is a hyperparameter controlling for the amount of shrinkage, with larger value indicating greater shrinkage.

When $\rho = 0$, Eq. (5) corresponds to LASSO, which sets a subset of θ to zero and is useful for variable selection. When $\rho = 1$, it represents Ridge regression, which shrinks all coefficient estimates toward zero but does not enforce exact zeros. Finally, when ρ is between 0 and 1, Eq. (5) becomes Elastic Net penalty, which serves as a compromise between Ridge and LASSO regularization techniques. Given that our full sample has 208 quarterly observations, we train the model on the full sample and select the optimal penalty parameter (λ , ρ) using cross validation.

2.2.2 Dimension Reduction: PCR and PLS

Equations Eq. (1) and Eq. (2) allow us to express the stock-bond correlation using matrix notation as:

$$Corr = Z\theta + E \tag{6}$$

where *Z* is a *T* × *P* matrix that contains the stacked characteristics and *E* is an vector of residuals ε_t . Given the relatively large number of characteristics (*P*), a practical way to deal with overfitting is through dimension reduction. Dimension reduction involves transforming a large set of variables into orthogonal components, effectively representing many variables with a smaller set of factors.

The principal component analysis (PCA) is one of the most commonly used dimension reduction methods. It identifies orthogonal components that capture the common variation across all characteristics. Then, a few leading components (K) serve as representatives of the characteristics and explain most of their variation. Apparently, PCA is an unsupervised method and it does not guarantee that these principal components are closely aligned with the best set of variables for explaining the target variable. Due to its ease of implementation, PCA has wide applications in all areas of science, including in particular management, finance, economics (Kelly, Pruitt, and Su, 2019; Kim, Korajczyk, and Neuhierl, 2020; Giglio and Xiu, 2021).

In contrast to PCA, Partial Least Square (PLS) directly associates characteristics with the target variable by regressing individual characteristics on the target variable in the first step. The resulting coefficients capture how sensitive the stock-bond comovement is to each individual characteristic. Therefore, PLS seeks *K* linear combinations of the *Z* matrix to maximize their covariance with the target variable. As a supervised method for variable selection and dimension reduction, PLS has been shown effective in reducing common noises and information aggregation (Kelly and Pruitt, 2013, 2015; Huang, Jiang, Tu, and Zhou, 2014; Huang, Li, and Wang, 2021). We train the model on the full sample and select the optimal number of dimension (*K*) using cross validation.

3 Stock-Bond Correlation

3.1 Stock vs. Treasury Bond

We commence by exploring the correlation between stock and ten-year Treasury return using both realized and MIDAS correlation. Panel A of Figure 1 reveals an intriguing trend: the correlation between stock and Treasury returns is positive prior to 1998 but undergoes a substantial shift towards negativity thereafter. Panel A of Table 3 shows the MIDAS correlation between stock and

Treasury stands at a positive and statistically significant level of 0.32 (with a *t*-statistic of 26.30) from 1969 Q1 to 1997 Q4. However, this correlation turns significantly negative starting from 1998 Q1 to 2020 Q4, registering at -0.30 (with a *t*-statistic of -13.44). Averaging the substantial positive and negative correlations across the full sample, we obtain a weak overall correlation of 0.05. This pattern is similarly observed when considering the realized correlation.

The striking shift suggests the possibility of a structural break occurring around 1997 to 1998. Therefore, we proceed to formally test this hypothesis. The results presented in Panel B of Table 3 indicate that, irrespective of whether we test for known, unknown, or multiple unknown breakpoints, the null hypothesis (No structural break) is consistently rejected at the 1% significance level. These results affirm the presence of a break time around 1998 Q1, which aligns with our conjecture. Overall, this shift in stock-Treasury correlation is consistent with the pattern that well documented in the literature (Baele, Bekaert, and Inghelbrecht, 2010; Campbell, Pflueger, and Viceira, 2020).

3.2 Stock vs. HY Corporate Bond

Differing from the existing studies that primarily focus on Treasury bonds, our research also explores the correlation between stocks and corporate bonds. Corporate bonds, characterized by their large market capitalization, are a prominent source of external financing for companies. Unlike Treasury bonds, which usually exhibit low credit risk, corporate bonds typically entail varying degrees of credit risk. IG corporate bonds generally possess higher credit ratings and lower default probabilities, resembling Treasury bonds in several aspects. Their correlation with stocks also mirrors that of Treasury bonds, as depicted in Figure B.1 in the Appendix B.

In contrast, HY corporate bonds, or junk bonds, present elevated credit risk and default probabilities, setting them apart from Treasury bonds. These "equity-like" corporate bonds may resemble stocks to some extent. Our study therefore extends to examine the correlation between stock and HY corporate bond. As expected, Panel B of Figure 1 reveals a consistent positive

correlation between stocks and HY corporate bonds from 1996 to 2020. After reaching an historical low (2002 Q3: 0.08) at the beginning of the 21st century, the correlation experienced an upward trend until the end of the sample period (2020 Q4: 0.59). This starkly contrasts with the notably negative correlation observed between stocks and Treasury bonds during this period. To be precise, we repeat the structural break test to detect if there exists any break time. Panel B of Table 3 demonstrates stock-HY correlation shares the same break time as stock-Treasury bond, which is 1998 Q1. The MIDAS correlation stands at 0.42 from 1994 Q2 to 1997 Q4, while slightly decrease to 0.32 from 1998 Q1 onwards, averaging for a correlation of 0.33 for the full sample.

These results raise interesting questions that motivate our study. Among the myriad characteristics documented in the literature, which ones exert the most significant influence on the stock-Treasury correlation as well as the stock-HY correlation? Do the characteristics that affect the stock-Treasury correlation differ from those influencing the stock-HY correlation? We strive to answer these questions utilizing advanced machine learning techniques.

[Insert Figure 1 and Table 3 about here]

4 Characteristic Importance

In this section, we evaluate the importance of individual candidate characteristic and use R-squared as the performance metric to assess their explanatory power on stock-bond correlations. We aim to identify the one with important influence on the stock-bond correlation while simultaneously controlling for the many others in the system. Specifically, for each method, we discover the importance of characteristic *j* by calculating the reduction in R-squared from setting all values of *j* to zero, while holding the remaining model estimates fixed (similar to the context of Gu, Kelly, and Xiu (2020)). We rank the total thirty characteristics and standardize each rank into the interval of [-1,1] following (Kelly, Pruitt, and Su, 2019). Then, we sum across five methods to get the overall standardized rank of each individual characteristic, which indicates its importance in explaining

the stock-bond correlation.

4.1 Stock vs. Treasury Bond

Previous research strives to identify the most influential factors that affect the correlation between stocks and Treasury bonds. However, the findings in this regard remain inconclusive. For instance, Connolly, Stivers, and Sun (2005) show that stock market uncertainty dominates this relationship, while Baele, Bekaert, and Inghelbrecht (2010) argue that liquidity proxies play a more critical role. More recently, Campbell, Pflueger, and Viceira (2020) demonstrate that the switch in correlation between inflation and the output gap explains the shift in stock-Treasury correlation from positive to negative. While most of these studies have taken a theoretical modeling approach, we adopt a different perspective, allowing the data to reveal its insights. We assess the significance of each potential characteristic using advanced machine learning techniques.

Figure 2 illustrates the overall importance of the thirty candidate characteristics, with the most influential ones at the top and the least impactful ones at the bottom. Across the entire sample period from 1969 Q1 to 2020 Q4, the five most influential characteristics are identified as: Real risk-free rate volatility (Vol_{RF}), Stock illiquidity ($Illi_{Stock}$), 10-year headline CPI change (*CPI10Y*), Macroeconomic uncertainty (UNC_{Macro}) and Risk aversion (*RiskAve*). Conversely, characteristics such as Industrial production uncertainty ($Unc_{IndProd}$), Credit Spread ($Firm_{CrSprd}$), and Idiosyncratic volatility ($Firm_{IdioVol}$) consistently rank lowest across almost all five approaches.

The results for full sample are intricate, as some characteristics contribute to positive stock-Treasury correlations while some may cause them to move in opposite directions. For instance, the stock illiquidity ($Illi_{Stock}$) is shown to be negatively and significantly related to stock and Treasury return comovement (Baele, Bekaert, and Inghelbrecht, 2010). This reflects the "flight to safety" phenomenon, where investors shift from the less liquid stocks to highly liquid Treasury bonds during periods of increased stock market illiquidity, inducing corresponding price changes. As a result, this leads to a negative correlation between stock and Treasury bond returns. On the other hand, the relationship between inflation as measured by CPI change (*CPI10Y*) and stock-Treasury correlation is less clear. While existing literature generally agrees that increasing inflation leads to lower bond returns, the effect on stock returns is somewhat inconclusive. On one hand, the nominal income from dividend payments increases with rising inflation. However, higher inflation also implies higher discount rate due to inflation illusion (Modigliani and Cohn, 1979; Campbell and Vuolteenaho, 2004). Hence, whether higher level of inflation leads to higher stock returns are still under debate.

Risk aversion (*RiskAve*) also has mixed effects. Theoretical work, such as Bekaert, Engstrom, and Grenadier (2010) and Wachter (2006), suggests that a rise in risk aversion may increase the real interest rate through a consumption smoothing effect or decrease it through a precautionary savings effect. As a consequence, the impact of risk aversion on stock-Treasury correlation remains unclear due to its ambiguous effects on interest rate. The impact of illiquidity is also unclear since existing literature suggests that its effect depends on how liquidity shocks comove across markets and the commonality in stock and Treasury bond liquidity are somewhat inconclusive as to which effect dominates (Chordia, Sarkar, and Subrahmanyam, 2005). In sum, the results for the full sample paint a complex picture with no clear-cut determination of the factors influencing stock-Treasury correlations. This complexity motivates us to conduct a deeper analysis, examining scenarios of positive and negative relationships separately.

[Insert Figure 2 about here]

4.1.1 Pre-break Period: Positive Correlation Scenario

Starting from the period from 1969 Q1 to 1997 Q4, characterized by a positive stock-Treasury correlation, we repeat the characteristics importance analysis using five machine learning techniques. As shown in Figure 3, top-ranking characteristics for the positive correlation scenario differ significantly from those observed in the full sample.

One noticeable finding is that among the top five high-ranking characteristics, three are

inflation-related measures. The most important one is the unemployment rate (G_{unemp}) which, according to the Phillips curve, is negatively correlated with the level of inflation. The second is 10-year headline CPI change (*CPI*10*Y*), which is a commonly-used measure of inflation. Additionally, the correlation between inflation and industrial production (*Cor_{Infl-IndProd}*) is also influential. As suggested by Campbell, Pflueger, and Viceira (2020), the correlation between inflation and industrial production to 1998, indicating that nominal bond prices decline in period of high marginal utility and they are risky assets. Because nominal bond returns are inversely related to inflation, our results demonstrate that during the period with a high level of inflation prior to 1998, Treasury bonds are perceived as risky assets that resemble stocks. They suffer from a "flight to safety" along with stocks and contribute to the positive stock-Treasury correlation.

In existing literature, the impact of inflation on stock-Treasury correlation is still inconclusive. While studies generally agrees that increasing inflation leads to lower bond returns, the effect on stock returns is somewhat inconclusive. On one hand, the nominal income from dividend payments increases with rising inflation. However, higher inflation also implies higher discount rate due to inflation illusion (Modigliani and Cohn, 1979; Campbell and Vuolteenaho, 2004). Our findings confirm that inflation plays a crucial role in the positive correlation scenario, which provide more cohesive evidence from a machine learning perspective.

Additionally, bond illiquidity ($Illi_{Bond}$) emerges as the second important characteristics for the period prior to 1998. This finding suggests that when Treasury bonds become less liquid and more risky, they resemble stocks more closely and both suffer from "flight to safety". Importantly, since both inflation and bond illiquidity profoundly impact bond returns, our findings demonstrate that it is the Treasury bond market fluctuations that primarily drive the stock-Treasury comovement for the pre-break period prior to 1998.

[Insert Figure 3 about here]

4.1.2 Post-break Period: Negative Correlation Scenario

We now shift our focus to the post-break period and repeat the characteristic importance analysis using five machine learning approaches from 1998 to 2020. Figure 4 reveals intriguing results that contrast sharply with those observed before 1998. The most influential characteristic is stock volatility (Vol_{SP}) which is much less important in the pre-break period. This suggests that increasing stock market volatility compels investors to shift their investments from stocks to Treasury bonds, leading to a negative stock-Treasury return correlation (Connolly, Stivers, and Sun, 2005). Furthermore, firm leverage ratio emerges as the second most important characteristic for post-break period, a significant departure from its relatively minor role before 1998. This underscores the substantial contribution of firm leverage to the negative stock-Treasury correlation. A higher leverage ratio implies a greater proportion of debt issuance relative to total assets, signaling deteriorating fundamentals and an increased likelihood of default. Consequently, investors tend to seek Treasury bonds as safe heaven to hedge against the increasing credit risk. This "flight-to-safety" phenomenon contributes to the negative stock-Treasury correlation.

On top of that, stock illiquidity (*Illi_{Stock}*) also plays a significant role in shaping the negative correlation scenario during the post-break period, although its contribution to the positive relationship observed before 1998 was comparatively less pronounced. Existing research suggests that the impact of liquidity on stock-Treasury comovements depends on the comovement of liquidity shocks across markets, and the consensus on the commonality in stock and Treasury bond liquidity is somewhat inconclusive (Chordia, Sarkar, and Subrahmanyam, 2005). Our results offers a clear perspective showing that it is stock market illiquidity that compels investors to shift from less liquid stocks to more liquid Treasury bonds. This shift induces price-pressure effects that may lead to negative stock-Treasury correlations. Consequently, our research adds valuable insights to resolve this puzzle from a machine learning standpoint.

Significantly, the characteristics with the most pronounced influence on the negative correlation scenario all relate to the phenomenon of cross-market hedging. Increased stock volatility, higher

firm leverage, stock illiquidity and elevated uncertainty collectively motivate investors to migrate from risky assets (stocks) to safer options (Treasury bonds). Treasury bonds serve as a secure hedge against the heightened levels of risk and uncertainty, resulting in a negative return correlation.

[Insert Figure 4 about here]

4.1.3 Rolling Window Analysis and Change in Characteristic Sign

We continue to explore the comparison between the pre-break and post-break periods by employing a LASSO method with a rolling window spanning 16 years. Our analysis ranks characteristics in descending order based on the difference in cumulative ranks before and after the year 2000. The most influential characteristics after 2000 are presented at the top, while those more impactful before 2000 are at the bottom. Results are reported in Figure 5. Prior to the year 2000, inflationrelated measures emerge as significant drivers of stock-Treasury correlation. Notably, the 10year headline CPI change (*CPI10Y*) and the unemployment rate (G_{unemp}) are ranked as the most influential characteristics during this period. These results align with previous findings that higher inflation levels lead to Treasury bonds being perceived as risky assets akin to stocks. Consequently, they experience a flight to safety in conjunction with stocks during this period, resulting in a positive stock-Treasury correlation.

Conversely, for the period following 2000, stock illiquidity and uncertainty measures ($Illi_{stock}$, Unc_{Macro}) take the spotlight as the more impactful characteristics. This further underscores the prominence of the cross-market hedging phenomenon in shaping stock-Treasury correlations during this time frame. In summary, the results presented in Figure 5 validate prior findings and offer a continuous perspective on the evolving significance of characteristics in stock-Treasury bond correlation over time. This analysis highlights the changing nature of the characteristics influencing stock-Treasury correlations, with inflation-related factors playing a more prominent role in the earlier period and cross-market hedging taking precedence in the later period.

[Insert Figure 5 about here]

We now delve into a detailed examination of the sign changes in characteristics, with a particular focus on the magnitude of these sign shifts before and after the break time. Characteristics are ranked in descending order based on the magnitude of their sign changes, with those exhibiting the most pronounced alterations placed at the top and the least pronounced ones at the bottom. Results are summarized in Figure 6 and several observations can be made. First, stock volatility (Vol_{SP}) and firm market leverage ($Firm_{Lev}$) emerge as the two characteristics with the most substantial sign changes. Upon closer investigation, it becomes evident that their coefficients were positive before the break but turned significantly negative for the post-break period. This supports the notion that they play pivotal roles in the negative correlation scenario but are of marginal significant during the pre-break period. Macroeconomic uncertainty (Unc_{Macro}) is another noteworthy characteristic, demonstrating importance in both the pre-break and postbreak periods, with coefficients changing from negative to positive. This highlights the idea that heightened uncertainty encourages increased cross-market hedging and results in a negative stock-Treasury correlation. Finally, bond illiquidity also exhibit a significant magnitude of sign change, consistent with its crucial role in the period before 1997 but its reduced significance in the later period. It's worth noting that our discussion does not focus on the sign of individual characteristics, as many of them exhibit inconclusive signs with respect to the stock-Treasury correlation (Baele, Bekaert, and Inghelbrecht, 2010; Chordia, Sarkar, and Subrahmanyam, 2005). Instead, our objective is to provide further evidence emphasizing the disparities in selected determinants before and after the break point, further enhancing the robustness of our earlier findings.

[Insert Figure 6 about here]

4.2 Stock vs. HY Corporate Bond

Having discussed the stock-Treasury correlation, we shift to the correlation between stock returns and HY corporate bonds. As observed in Figure 1 and detailed in Section 3.2, stock returns exhibit a consistently increasing positive correlation with HY corporate bond returns over the entire sample period. This positive relationship prompts us to examine whether the determinants of this correlation differ from those affecting the stock-Treasury correlation. We repeat the machine learning analysis, this time using the stock-HY correlation as the target.

As Figure 7 presents the results of this analysis, highlighting the firm's leverage ratio as the primary determinant of the correlation between stocks and HY corporate bonds. Both operating leverage and market leverage are consistently identified as top-ranking characteristics across all five machine learning methods. As a proxy of default risk, firm leverage is a critical concern for investors, as it indicates the level of financial instability and default probability. Given that stocks and corporate bonds are both claims on firms' assets, increasing credit risk and deterioration in firms' fundamentals prompt investors to demand higher compensation for holding these securities issued by highly leveraged companies. This also implies that for assets with substantial credit risk, investors are more concerned about the fundamentals of the issuing firms than macroeconomic conditions, such as inflation risk (Li, 2022).

The relationship between default risk and stock-bond correlation remains less conclusive in the existing literature. For instance, the Merton model claims that the spot correlation is invariant with firm default risk. With stochastic asset variance and interest rates, higher default risk leads to increased exposures of stocks and bonds to changes in asset value, thereby increasing stock-bond correlation. However, it also causes the product of stock and bond exposures to asset variance and interest rates to become more negative, which reduces the stock-bond correlation. These two opposing forces makes the impact of default risk on stock-bond correlation less intuitive (Dickerson, Fournier, Jeanneret, and Mueller, 2022). Our study, on the other hand, reveals that firm leverage and default risk play a crucial role in explaining the correlation between stock and HY

corporate bonds. Utilizing advanced machine learning techniques, we allow data to tell the truth, which provides more comprehensive and coherent evidences complementing existing research.

Furthermore, macroeconomic uncertainty (Unc_{Macro}) plays a vital role in influencing the stock-HY correlation. This suggests that HY corporate bonds share characteristics with stocks as risky assets, and when economic uncertainty increases, both asset classes experience a "flight to safety". Lastly, due to their lower liquidity, HY corporate bonds are more sensitive to overall liquidity risk in the bond market. This phenomenon highlights the importance of market liquidity as a contributing factor to the correlation. In summary, these results emphasize that investors pay more attention to credit risk and the financial health of issuing firms when dealing with risky assets. As a result, they demand greater compensation for holding these assets, leading to the positive correlation observed between stock and HY corporate bonds.

4.3 Treasury vs. HY Corporate Bond

In the last part of this section, we discuss a bit on the relationships between Treasury bonds and HY corporate bonds. Our earlier analysis demonstrated that while the stock-Treasury correlation shifts from positive to negative, the stock-HY correlation remains consistently positive throughout the sample period. Given that Treasury bonds and HY corporate bonds share certain structural similarities but are issued by different entities, their correlation could potentially resemble either of the two scenarios mentioned above. On one hand, they both are influenced by characteristics that primarily impact bonds, such as inflation and bond market illiquidity. On the other hand, HY corporate bonds carry a higher degree of credit risk and default probability compared to Treasury bonds. Hence, they are more susceptible to the fundamentals of the issuing firms. Figure 9 illustrates that the correlation between Treasury bonds and HY corporate bonds is generally positive in the period before 2008, but shifts towards negativity thereafter, resulting in an overall decreasing trend. This correlation pattern resembles that of stocks and Treasury bonds, suggesting that HY corporate bonds indeed exhibit characteristics similar to equities.

Next, we repeat the characteristic importance analysis using the Treasury-HY correlation as the target variable. Figure 9 reveals both inflation and firm leverage ratios are selected as influential characteristics in driving the Treasury-HY correlation. Consistent with discussions earlier, during periods of elevated inflation, Treasury bonds are perceived as risky assets and resemble HY corporate bonds, resulting in a positive correlation. Conversely, when inflation returns to normal levels, Treasury bonds are viewed as a hedge against the credit risk and default probability associated with HY corporate bonds.

It's necessary to note that our analysis here does not aim to delve further into the specific patterns and determinants of the Treasury-HY correlation, as it falls beyond the scope of this paper. Instead, our intention is to emphasize that Treasury bonds can serve as a safe hedge against both stocks and HY corporate bonds during periods of low inflation. This potential diversification benefit can aid investors in risk management within their investment portfolios.

[Insert Figure 8 and 9 about here]

4.4 Economic Contributions

Our previous results provide a qualitative ranking about the importance of each characteristics in determining the stock-bond comovement. This prompts a further question that to what extent different characteristics quantitatively contribute to stock-bond correlation. To address this, we reestimate the model using the LASSO approach, leaving out certain characteristics, and calculate the deterioration in fit.

Specifically, for both realized (Real) and MIDAS (MD) correlations, we conduct the model selection tests including all characteristics and denote this as the baseline results (Baseline). We calculate the R-squared measures, the distance measures and correlation measures of the baseline model as benchmarks for the tests. Distance measures compute the mean absolute deviation (MAE) of the model-implied stock-bond correlations from respectively the MIDAS conditional

correlations and the realized correlations. Correlation measures compute the unconditional correlation between our model-implied conditional correlations and respectively the MIDAS conditional correlations and the realized correlations. Hence, Higher distance values indicate worse fit of the model compared to target, while higher correlation values indicate better fit. Then, we repeat the LASSO model selection tests setting certain characteristics to zero and compute the deterioration of the restricted model relative to the baseline model. For the R-squared measures, we compute the deterioration as $100 \times$ (R-squared baseline model - R-squared restricted model) / R-squared baseline model. For the distance measures, we compute the deterioration as $100 \times$ (MAE restricted model - MAE baseline model) / MAE baseline model. For the correlation measures, we report the difference between the correlation for the baseline model and the correlation for the restricted model, expressed in percentage.

We start from the results of stock-Treasury correlation, reported in the left panel of Table 4. First, the model selected by LASSO fits the stock-Treasury correlation well, with R-squared measures and correlation measures above 0.90 for MIDAS correlation. Second, the model fit deteriorates considerably more when macroeconomic variables are omitted compared to non-macroeconomic variables. For instance, considering MIDAS correlation, the R-squared measure decreases by approximately 2.5 times when dropping macroeconomic variables, compared to 1.7 times dropping non-macroeconomic variables. The decrease in correlation measures is more than doubled when macroeconomic variables are left out compared to non-macroeconomic variables.

Within the set of macroeconomic variables, inflation is the most significant contributor, followed by correlation variables across all three measures. Among the non-macroeconomic variables, volatility variables are the most influential contributors, closely followed by illiquidity measures. In contrast, firm characteristics and the variance premium overall contribute the least. In summary, these findings reaffirm our previous findings from a quantitative perspective. Inflation is the most significant factor driving stock-Treasury comovement, particularly in the positive correlation scenario, followed by illiquidity and volatility variables, which have a more pronounced

impact on negative stock-Treasury correlations.

The right panel of Table 4 shows the results for stock-HY correlation, which are in sharp contrast to those of stock-Treasury correlation, with non-macroeconomic variables contributing much more than macroeconomic variables. For example, considering the MIDAS correlation, the increment in distance measure when dropping non-macroeconomic variables is more than five times that when macroeconomic variables are omitted (237.58 vs. 45.95). Among the non-macroeconomic variables, firm characteristics (e.g., leverage, idiosyncratic volatility) have the most significant impact, followed by illiquidity and uncertainty measures. In comparison, macroeconomic variables, especially inflation and sentiment measures contribute the least. Overall, these results also corroborate the findings in Figure 7 that firm-level characteristics and illiquidity measures are the most influential determinants of the positive stock-HY correlation while macroeconomic characteristics have a lesser impact.

[Insert Table 4 about here]

4.5 Correlation Decomposition

In this section, we proceed to gain a deeper understanding of the MIDAS correlation by decomposing it into three components, including stock volatility, bond volatility, and covariance. To explore the contribution of top-ranking characteristics to these three components, we perform LASSO model selection tests including all characteristics, denoted as the baseline results (Baseline). Then, we repeat the LASSO tests while omitting specific characteristics and measure the deterioration in the restricted model's performance relative to the baseline model. This deterioration is expressed as the percentage difference in correlations between the baseline model and the restricted model.

Results in Panel A of Table 5 reveal that, for the full sample of stock-Treasury correlation, topranking characteristics have a relatively even impact on both stock volatility and bond volatility. However, when we narrow down to the pre-break period from 1969 Q1 to 1997 Q4, most of the characteristics significantly affect bond volatility compared to stock volatility. In contrast, for the post-break period from 1998 Q1 onwards, the selected characteristics exert a greater influence on the covariance component. These results reinforce the notion that the variation in stock-Treasury correlation during the pre-break period is primarily driven by Treasury bonds, influenced by factors such as inflation and illiquidity. In contrast, cross-market hedging explains most of the variation in the negative correlation scenario for the post-break period.

For the stock-HY correlation, as shown in the last panel of Table 5, results indicate that most of the top-ranking characteristics have a notable impact on the covariance component. This further confirms that for risky assets, such as stocks and HY corporate bonds, investors and lenders pay considerable attention to the fundamentals of issuing firms. Consequently, characteristics that influence the stock-HY correlation would impact both stocks and bonds in a balanced manner.

In the final part of our analysis, we perform multivariate regressions of top-ranking characteristics on the MIDAS correlation and its three components (covariance, stock volatility, and bond volatility). Once again, the results corroborate our previous findings in Table 5, further solidifying the robustness of our analysis.

[Insert Table 5 and Table 6 about here]

5 Conclusion

We study the comovements between stocks and bonds by focusing on Treasury bonds and corporate bonds separately. The stock-Treasury bond correlation transitions from positive to negative while the correlation between stocks and HY corporate bonds consistently remains positive displaying a notable increasing pattern. Employing advanced machine learning techniques and an extensive panel of macroeconomic characteristics, we identify key drivers for their correlations. For stock and Treasury bonds, inflation and bond illiquidity are the primary driving forces behind the positive correlation scenario, while the negative scenario is largely explained by the cross-market hedging phenomenon. Regarding stocks and HY corporate bonds, default risk and bond illiquidity emerge as crucial factors influencing their comovement. We conduct a series of empirical analysis to establish the robustness of our findings.

Our study connects to two major strands of literature. We revisit the stock-Treasury correlation from a machine learning perspective, offering a more comprehensive and objective resolution to the ongoing debates and arguments within the current literature. Our innovative exploration of the comovements between stocks and HY corporate bonds fills a significant gap in existing research, providing inspiration for future studies in this field. By applying machine learning techniques within the context of macroeconomics, we advance the utilization of machine learning in financial research and lay the groundwork for further investigations.





This figure plots the realized quarterly correlation and MIDAS correlation between stock and bond following Colacito, Engle, and Ghysels (2011). Panel A plots the correlation between stock and ten-year Treasury bond. Sample period is from 1969 Q1 to 2020 Q4. Panel B plots the correlation between stock and HY corporate bond. Sample period is from 1994 Q2 to 2020 Q4.



Figure 2 Characteristic importance in stock-Treasury correlation

This figure plots the importance of thirty characteristics in stock-Treasury correlation using five machine learning approaches, including Least Absolute Shrinkage and Selection Operator (LASSO), Ridge Regression (Ridge), Elastic Net Regularization (ENet), Principal Component Regression (PCR), and Partial Least Square Regression (PLS). Characteristics are ranked in descending order of their cumulative ranks across five models, with the most impactful characteristics at the top and the least impactful ones at the bottom. The detailed definition of characteristics is given in Table 1 and the summary statistics are shown in Table 2. Sample period is from 1969 Q1 to 2020 Q4.



Figure 3 Characteristic importance in stock-Treasury correlation: Positive scenario

This figure plots the importance of thirty characteristics in stock-Treasury correlation using five machine learning approaches, including Least Absolute Shrinkage and Selection Operator (LASSO), Ridge Regression (Ridge), Elastic Net Regularization (ENet), Principal Component Regression (PCR), and Partial Least Square Regression (PLS). Characteristics are ranked in descending order of their cumulative ranks across five models, with the most impactful characteristics at the top and the least impactful ones at the bottom. Sample period is from 1969 Q1 to 1997 Q4.



Figure 4 Characteristic importance in stock-Treasury correlation: Negative scenario

This figure plots the importance of thirty characteristics in stock-Treasury correlation using five machine learning approaches, including Least Absolute Shrinkage and Selection Operator (LASSO), Ridge Regression (Ridge), Elastic Net Regularization (ENet), Principal Component Regression (PCR), and Partial Least Square Regression (PLS). Characteristics are ranked in descending order of their cumulative ranks across five models, with the most impactful characteristics at the top and the least impactful ones at the bottom. Sample period is from 1998 Q1 to 2020 Q4.



Figure 5 Rolling window LASSO in stock-Treasury correlation

This figure plots the importance of thirty characteristics in stock-Treasury correlation using Least Absolute Shrinkage and Selection Operator (LASSO) with a rolling window of 16 years. Characteristics are ranked in descending order based on the difference of cumulative ranks prior to and post 2000, with the most impactful characteristics after 2000 at the top and the most impactful ones before 2000 at the bottom. Sample period is from 1969 Q1 to 2020 Q4.



Figure 6 Characteristic sign in stock-Treasury correlation

This figure plots the change in the sign of characteristics prior to and post 1997 Q4 using four machine learning approaches, including Least Absolute Shrinkage and Selection Operator (LASSO), Ridge Regression (Ridge), Elastic Net Regularization (ENet), and Partial Least Square Regression (PLS). Characteristics are ranked in descending order based on the magnitude of their changes in sign, with the characteristics exhibiting the most pronounced sign changes at the top and the least pronounced ones at the bottom. Sample period is from 1969 Q1 to 2020 Q4.



Figure 7 Characteristic importance in stock-HY correlation

This figure plots the importance of thirty characteristics in stock-HY bond correlation using five machine learning approaches, including Least Absolute Shrinkage and Selection Operator (LASSO), Ridge Regression (Ridge), Elastic Net Regularization (ENet), Principal Component Regression (PCR), and Partial Least Square Regression (PLS). Characteristics are ranked in descending order of their cumulative ranks across five models, with the most impactful characteristics at the top and the least impactful ones at the bottom. Sample period is from 1994 Q2 to 2020 Q4.



Figure 8 Treasury-HY correlation

This figure plots the realized quarterly correlation and MIDAS correlation between ten-year Treasury and ICE high yield corporate bond index returns following Colacito, Engle, and Ghysels (2011). Sample period is from 1994 Q2 to 2020 Q4.



Figure 9 Characteristic importance in Treasury-HY correlation

This figure plots the importance of thirty characteristics in Treasury-HY correlation using five machine learning approaches, including Least Absolute Shrinkage and Selection Operator (LASSO), Ridge Regression (Ridge), Elastic Net Regularization (ENet), Principal Component Regression (PCR), and Partial Least Square Regression (PLS). Characteristics are ranked in descending order of their cumulative ranks across five models, with the most impactful characteristics at the top and the least impactful ones at the bottom. Sample period is from 1994 Q2 to 2020 Q4.

Table 1Characteristics definitions

This table reports the details of the thirty fundamental characteristics from the United States used in the analysis. Sample period is from 1969 Q1 to 2020 Q4.

Variable	Label	Description
Output Gap	Out put Gap	Percentage difference between output and its quadratic trend.
Risk Aversion	RiskAve	Time-varying relative risk aversion coefficient of the representative agent
		in a generalized habit-like model from Bekaert, Engstrom, and Xu (2022).
Variance Risk Premium	VRP	The VIX squared minus fitted MIDAS variance.
Investor Sentiment	SENT	The investor sentiment index from Baker and Wurgler (2006).
10-year CPI Change	CPI10Y	Trailing 10-year annualised changes in headline CPI.
1-year CPI Change	CPI1Y	One-year change in headline CPI.
Cash Flow Growth	G _{Cash}	S&P 500 dividend growth including repurchases.
Industrial Production Growth	G _{IndProd}	One-year change in US real industrial production.
Non-farm Payroll Growth	G _{Payroll}	Natural logarithm of non-farm payrolls.
Unemployment Rate	G_{Unemp}	U-3 Unemployment rate.
Corporate Profit Growth	<i>G_{CorpProfit}</i>	Natural logarithm of corporate profits after tax.
Real Federal Funds Rate	R _{RealFed}	Effective rate minus 12-month core PCE Price Index.
Monetary Policy Gap	R _{TaylorRuleGap}	Monetary policy gap (versus Taylor rule).
Monetary Policy Gap	$R_{NeuralRateGap}$	Monetary policy gap (versus Neutral Rate).
Inflation-Output Cor.	Cor _{Infl-Output}	Rolling 10-year correlation between inflation and output growth.
Inflation-Output Gap Cor.	Cor _{Infl-OutputGap}	Rolling 10-year correlation between inflation and output gap.
Inflation-Industrial Production Cor.	Cor _{Infl-IndProd}	Rolling 10-year correlation between inflation and industrial production.
Stock Volatility	Vol_{SP}	One-year annualised standard deviation of S&P 500 total returns.
Real Risk-free Rate Volatility	Vol_{Rf}	Trailing 5-year annualised standard deviation of monthly change in short- term real yields.
Macro Uncertainty	Unc_{Macro}	Macroeconomic uncertainty from Jurado, Ludvigson, and Ng (2015).
Output Uncertainty	Unc _{Out put}	Mean variance of one quarter-ahead real GDP growth and dispersion of
	1	real GDP growth over the next four quarters from SPF.
Inflation Uncertainty	Unc _{Infl}	Mean variance of one year-ahead inflation and dispersion of inflation over
	U	the next four quarters from SPF.
Industrial Production Uncertainty	Unc _{IndProd}	SPF Industrial production forecast dispersion.
Unemployment Uncertainty	Unc _{Unemp}	SPF Unemployment rate forecast dispersion.
Stock Illiquidity	Illi _{Stock}	Capitalization-based proportion of zero daily returns across all firms.
Bond Illiquidity	Illi _{Bond}	Monthly equally weighted average of quoted spreads across all securities.
Market Leverage	<i>Firm_{Lev}</i>	Total liabilities divided by market value of equity.
Operation Leverage	<i>Firm_{OpLev}</i>	Sum of administrative expenses and cost of goods, scaled by total assets.
Idiosyncratic Risk	Firm _{IdioVol}	Standard deviation of residuals from Fama-French three factor regressions
		using the past month of daily data.
Credit Spread	Firm _{CrSprd}	The difference between BAA and AAA-rated corporate bond yields.

Table 2	Summary	statistics
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This table reports the summary statistics of thirty characteristics used in the analysis. Sample period is from 1969 Q1 to 2020 Q4.

Variable	Obs	Mean	SD	Min	P25	Median	P75	Max	Skew	Kort
Out put Gap	208	-0.004	0.036	-0.105	-0.025	-0.008	0.015	0.073	0.173	0.166
RiskAve	208	3.008	0.717	2.355	2.574	2.770	3.085	6.230	2.497	6.664
VRP	208	-0.469	0.517	-4.487	-0.565	-0.333	-0.194	0.021	-4.494	28.054
SENT	208	0.020	0.958	-2.079	-0.405	-0.084	0.583	3.053	0.521	1.242
CPI10Y	208	0.040	0.021	0.014	0.025	0.033	0.057	0.088	0.861	-0.529
CPI1Y	208	0.040	0.029	-0.014	0.021	0.031	0.049	0.146	1.479	2.192
G _{Csah}	208	0.004	0.034	-0.139	-0.014	0.005	0.025	0.104	-0.531	1.611
G _{IndProd}	208	0.019	0.047	-0.152	-0.001	0.025	0.049	0.118	-0.897	1.538
<i>G</i> _{Payroll}	208	0.003	0.009	-0.092	0.002	0.004	0.007	0.030	-6.582	71.511
G_{Unemp}	208	0.062	0.017	0.034	0.050	0.059	0.073	0.110	0.673	-0.055
GCorpProfit	208	0.017	0.073	-0.530	-0.009	0.020	0.045	0.406	-1.247	19.222
R _{RealFed}	208	0.017	0.026	-0.044	-0.006	0.017	0.037	0.102	0.409	-0.310
R _{TaylorRuleGap}	208	-0.004	0.027	-0.095	-0.022	-0.008	0.017	0.066	0.088	0.211
R _{NeuralRateGap}	208	-0.008	0.021	-0.074	-0.022	-0.012	0.005	0.069	0.424	1.190
Cor _{Infl-Output}	208	-0.181	0.411	-0.723	-0.493	-0.290	0.076	0.653	0.569	-0.876
Cor _{Infl-OutputGap}	208	0.193	0.379	-0.612	-0.123	0.296	0.501	0.812	-0.380	-1.031
Cor _{Infl-IndProd}	208	-0.144	0.410	-0.742	-0.506	-0.225	0.068	0.664	0.428	-1.059
Volsp	208	0.041	0.016	0.011	0.028	0.040	0.050	0.088	0.688	0.475
Vol_{Rf}	208	0.002	0.001	0.001	0.001	0.002	0.003	0.006	1.466	1.462
Unc _{Macro}	208	0.797	0.109	0.668	0.730	0.767	0.823	1.244	1.738	3.220
Unc _{Out put}	208	-0.004	0.268	-0.272	-0.200	-0.070	0.079	1.414	1.913	4.578
Unc _{Infl}	208	-0.038	0.633	-0.910	-0.471	-0.252	0.237	2.625	1.715	3.339
Unc _{IndProd}	208	0.048	4.026	-3.762	-2.846	-1.376	1.460	19.137	1.854	4.053
Unc _{Unemp}	208	-0.013	0.624	-0.497	-0.288	-0.097	0.103	7.691	9.446	113.486
Illi _{Stock}	208	0.076	0.060	0.001	0.012	0.099	0.127	0.180	0.011	-1.671
Illi _{Bond}	208	0.133	0.112	0.030	0.037	0.066	0.237	0.399	0.731	-1.032
<i>Firm_{Lev}</i>	208	1.671	0.495	0.531	1.363	1.684	1.986	3.113	0.027	0.010
<i>Firm_{OpLev}</i>	208	0.750	0.145	0.513	0.618	0.740	0.876	1.061	0.106	-1.324
Firm _{IdioVol}	208	0.015	0.005	0.007	0.011	0.014	0.016	0.035	1.275	1.985
Firm _{CrSprd}	208	0.011	0.004	0.005	0.008	0.010	0.013	0.033	1.876	5.097

Table 3Structural break test

This table reports the break time of the realized correlation and MIDAS correlation between stock and bond via multiple structural break test. Panel A reports the means of the realized correlation and MIDAS correlation for the entire sample period and the sample period prior to and post 1997, with t-test values in parentheses. Panel B presents multiple structural break test results and the times at which structural breakpoints occurred for the MIDAS correlation.

Panel A: Stock-Treasury Correlation			
	Full Sample	1969Q1-1997Q4	1998Q1-2020Q4
Realized Correlation	0.0426	0.3225	-0.3104
	(1.53)	(16.18)	(-10.33)
MIDAS Correlation	0.0483	0.3220	-0.2967
	(1.98)	(26.30)	(-13.44)
	Break Time	H_0	<i>p</i> -Value
Test for a Known Breakpoint	1998Q1	No Structural Break	***
Test for an Unknown Breakpoint	1998Q1	No Structural Break	***
Test for Multiple Unknown Breakpoints	1998Q1	No Structural Break	***
Cumulative Sum Test for Parameter Stability	/	No Structural Break	***
Panel B: Stock-HY Bond Correlation			
	Full Sample	1994Q2-1997Q4	1998Q1-2020Q4
Realized Correlation	0.3347	0.4220	0.3205
	(17.26)	(8.53)	(15.42)
MIDAS Correlation	0.3313	0.4300	0.3152
	22.97	17.50	20.09
	Break Time	H_0	<i>p</i> -Value
Test for a Known Breakpoint	1998Q1	No Structural Break	***
Test for an Unknown Breakpoint	1998Q1	No Structural Break	***
Test for Multiple Unknown Breakpoints	1998Q1	No Structural Break	***
Cumulative Sum Test for Parameter Stability	/	No Structural Break	***

Table 4Economic contributions

This table reports the economic contributions of the different characteristics for the realized (Real) and MIDAS (MD) correlations. For both correlations, we repeat the results of the model selection tests of the model including all characteristics (Baseline). Next, we compute the model selection test measures by setting certain factors to zero. We differentiate between leaving out macro factors and leaving out non-macro factors. For each measure we compute the deterioration of the restricted model relative to the baseline model. For the R-squared measures, we compute the deterioration as $100 \times$ (R-squared baseline model - R-squared restricted model)/R-squared baseline model. For the distance measures, we compute the deterioration as $100 \times$ (MAE restricted model - MAE baseline model)/MAE baseline model. For the correlation measures, we report the difference between the correlation for the baseline model and the correlation for the restricted model, multiplied by one hundred. All numbers are expressed in percentages. The stock-Treasury bond correlation sample period is from 1969 Q1 to 2020 Q4. The stock-HY bond correlation sample period is from 1994 Q2 to 2020 Q4.

		Stoc	k-Treas	sury Correlation Stock				-HY Bond Correlation				
	R-s	quared	Dis	stance	Corre	elation	R-sc	luared	Dis	stance	Corre	elation
LASSO Model	MD	Real	MD	Real	MD	Real	MD	Real	MD	Real	MD	Real
Baseline	0.92	0.68	0.08	0.19	0.96	0.83	0.73	0.34	0.06	0.12	0.86	0.61
Minus Macro Var	232.58	15.97	448.68	14.78	49.64	3.45	34.18	41.62	45.95	13.70	10.47	3.57
Inflation var	85.67	0.00	253.16	0.00	7.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Growth var	1.10	0.00	8.22	0.00	0.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate var	6.65	13.80	35.33	12.84	1.38	3.03	3.58	0.00	5.84	0.00	1.26	0.00
Correlation var	17.15	0.78	79.54	0.57	3.64	0.07	4.94	4.71	7.86	0.91	1.10	0.59
Out put Gap	2.98	0.00	13.87	0.00	1.37	0.00	23.48	33.58	37.09	11.88	4.13	1.41
SENT	0.36	0.00	3.25	0.00	0.16	0.00	0.50	0.00	0.75	0.00	0.19	0.00
Minus Non-macro Var	177.87	93.24	419.24	80.16	15.58	22.24	278.48	221.51	237.58	56.89	16.02	7.68
Uncertainty var	27.49	0.00	119.98	0.00	1.34	0.00	187.07	0.00	186.80	0.00	2.22	0.00
Volatility var	67.37	0.00	215.75	0.00	5.56	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Illiquidity var	26.19	144.06	110.59	106.85	2.29	17.59	279.54	189.19	238.05	53.64	7.61	5.99
Firm var	0.76	0.00	4.65	0.00	0.00	0.00	2041.00	712.28	846.27	152.63	11.42	5.69
RiskAve	21.54	18.16	99.90	17.74	0.57	1.09	0.00	0.00	0.00	0.00	0.00	0.00
VRP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 5Top characteristics contribution

This table reports the economic contribution of the top characteristics. The MIDAS correlation is decomposed into three components: covariance, stock volatility, and bond volatility. We select the top five variables of importance identified by machine learning analysis, setting factors to zero one by one and computing the correlation measure between the restricted model and the baseline model. For the correlation measures, we report the difference between the correlation for the baseline model and the correlation for the restricted model, multiplied by one hundred. Panel A reports results for the full sample period of stock-Treasury correlation from 1969 Q1 to 2020 Q4, while Panel B reports results for the pre-break period of stock-Treasury correlation from 1969 Q1 to 1997 Q4, Panel C reports results for the post-break period of stock-Treasury correlation from 1998 Q1 to 2020 Q4. Panel D reports results for the full sample period of stock-HY bond correlation from 1994Q2 to 2020 Q4.

	MIDAS Correlation	Covariance	Stock Volatility	Bond Volatility
Panel A: Full Sar	mple of Stock-Treasury	Correlation		
Baseline	0.96	0.95	0.96	0.86
Vol_{Rf}	5.52	5.56	0.00	0.00
CPI10Y	4.09	9.95	2.09	8.80
Illi _{Stock}	4.09	0.44	0.00	5.88
Unc _{Macro}	0.24	1.03	4.53	1.37
RiskAve	0.57	0.23	0.00	0.46
Panel B: Pre-brea	ak 1969Q1-1997Q4			
Baseline	0.90	0.93	0.94	0.92
G_{Unemp}	2.00	6.02	0.00	0.00
Illi _{Bond}	53.13	22.94	0.27	6.73
Unc _{Macro}	0.09	0.00	0.14	12.35
CPI10Y	2.31	20.96	0.53	11.92
Cor _{Infl-IndProd}	5.06	5.18	0.00	0.59
Panel C: Post-bre	eak 1998Q1-2020Q4			
Baseline	0.93	0.95	0.98	0.96
Vol _{SP}	4.77	4.58	0.21	1.46
<i>Firm_{Lev}</i>	0.70	1.05	0.14	0.00
Illi _{Stock}	2.12	0.40	0.00	0.98
Unc _{Macro}	0.36	0.00	0.44	0.00
Out put Gap	2.02	0.60	0.01	0.07
Panel D: Full Sar	mple Stock-HY Bond (Correlation		
Baseline	0.86	0.87	0.98	0.94
<i>Firm_{OpLev}</i>	9.37	7.81	0.00	0.00
Unc _{Macro}	2.52	6.30	0.59	6.39
Illi _{Bond}	7.61	6.27	0.00	0.37
<i>Firm_{Lev}</i>	1.16	1.24	0.00	0.00
Out put Gap	4.13	6.13	0.00	1.22

Table 6Top characteristics regression

This table reports the regression analysis of the top characteristics. The MIDAS correlation is decomposed into three components: covariance, stock volatility, and bond volatility. We select the top five variables of importance identified by machine learning analysis as explanatory variables. Panel A reports results for the full sample period of stock-Treasury correlation from 1969 Q1 to 2020 Q4, while Panel B reports results for the pre-break period of stock-Treasury correlation from 1969 Q1 to 1997 Q4, Panel C reports results for the post-break period of stock-Treasury correlation from 1998 Q1 to 2020 Q4. Panel D reports results for the full sample period of stock-HY bond correlation from 1994Q2 to 2020 Q4. Standard errors are in parentheses. *, **, and *** denotes significance at the 10%, 5%, and 1% level, respectively

М	IDAS Correlation	Covariance St	ock Volatility	Bond Volatility
Panel A: Full Sample of Sto	ock-Treasury Cor	relation		
$\overline{Vol_{Rf}}$	-0.451***	-0.458***	-0.095	0.055
	(0.053)	(0.056)	(0.081)	(0.084)
CPI10Y	0.600***	0.798***	-0.297^{**}	0.621***
	(0.083)	(0.087)	(0.125)	(0.129)
Illi _{Stock}	0.541***	0.354***	-0.007	-0.500^{***}
	(0.062)	(0.066)	(0.094)	(0.097)
Unc _{Macro}	-0.063	-0.116^{***}	0.369***	0.147*
	(0.039)	(0.041)	(0.059)	(0.060)
RiskAve	-0.051	-0.083^{*}	0.494***	0.296***
	(0.040)	(0.042)	(0.061)	(0.063)
R-squared	0.81	0.79	0.56	0.53
Number of observations	208	208	208	208
Panel B: Pre-break 1969Q1	-1997Q4			
G _{Unemp}	-0.137***	-0.098^{**}	-0.059	0.120
	(0.036)	(0.040)	(0.078)	(0.120)
Illi _{Bond}	-0.434^{***}	-0.443^{***}	-0.050	-0.663***
	(0.045)	(0.050)	(0.098)	(0.152)
Unc _{Macro}	0.097**	0.193***	0.177**	0.913***
	(0.039)	(0.043)	(0.084)	(0.130)
CPI10Y	0.186***	0.239***	0.078	0.479***
	(0.038)	(0.042)	(0.082)	(0.127)
Cor _{Infl-IndProd}	-0.130^{**}	-0.082^{**}	0.059	0.111
	(0.064)	(0.072)	(0.140)	(0.217)
R-squared	0.61	0.56	0.09	0.56
Number of observations	208	208	208	208

Table 6
Continued

Panel C: Post-break 1998Q	1-2020Q4			
Vol _{SP}	-0.178***	-0.216***	0.455***	0.279***
	(0.035)	(0.033)	(0.072)	(0.047)
<i>Firm_{Lev}</i>	-0.224^{***}	-0.354***	0.396***	0.448^{***}
	(0.035)	(0.038)	(0.082)	(0.053)
Illi _{Stock}	2.012***	1.540***	1.208**	0.921**
	(0.035)	(0.264)	(0.576)	(0.373)
Unc _{Macro}	0.147***	0.048	0.619***	0.077^{*}
	(0.035)	(0.032)	(0.070)	(0.045)
Out put Gap	0.240***	0.237***	-0.149**	-0.018
	(0.035)	(0.034)	(0.074)	(0.048)
R-squared	0.81	0.86	0.82	0.72
Number of observations	208	208	208	208
Panel D: Full Sample of Sto	ock-HY Bond C	orrelation		
Firm _{OpLev}	-0.378***	-0.346***	0.281***	0.072
	(0.064)	(0.062)	(0.059)	(0.062)
Unc _{Macro}	0.157**	0.281***	0.683***	0.737***
	(0.064)	(0.062)	(0.059)	(0.062)
Illi _{Bond}	0.510***	0.459***	-0.409^{***}	-0.034
	(0.085)	(0.082)	(0.078)	(0.081)
<i>Firm_{Lev}</i>	-0.077	-0.091	0.051	0.135 * *
	(0.065)	(0.063)	(0.060)	(0.063)
Out put Gap	-0.337***	-0.389***	-0.396***	-0.285^{***}
	(0.079)	(0.076)	(0.072)	(0.076)
R-squared	0.69	0.71	0.74	0.71
Number of observations	107	107	107	107

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Appendix

A DCC-MIDAS Model

The DCC-MIDAS model is a multivariate extension to the GARCH-MIDAS model with dynamic correlations. Following Colacito, Engle, and Ghysels (2011), we estimate the long-run stock-bond correlation in two steps. In the first step, we estimate conditional stock and bond return variances using univariate GARCH-MIDAS models separately. In the second step, returns are standardized by subtracting the estimated means and dividing by the conditional variances to obtain the standardized residuals. We can calculate condition stock-bond correlations based on these standardized residuals.

A.1 GARCH-MIDAS Model

Following Engle, Ghysels, and Sohn (2013) univariate GARCH-MIDAS model framework, we assume the asset return $r_{i,t}$ of day *i* in quarter *t* (either stock return or bond return in the context) follows the GARCH-MIDAS process:

$$r_{i,t} = \mu + \sqrt{\tau_t g_{i,t}} \xi_{i,t}, \forall i = 1, ..., N_t$$

$$\xi_{i,t} | \varphi_{i-1,t} \sim N(0,1)$$
(A.1)

where N_t denote the number of trading days in the quarter t and $\varphi_{i-1,t}$ is the information set up to day (t-1) of quarter t. In the Equation(A.1), the conditional return variance is decomposed into the short-run component $g_{i,t}$, which varies at the daily frequency and the long-run component τ_t , which only changes every quarter t. The short-run component $g_{i,t}$ follows a simple GARCH(1,1) processs, while the long-run component τ_t is the weighted sum of quarterly realized variance RV_{t-k} over last K_v quarters.

$$g_{i,t} = (1 - \alpha - \beta) + \alpha \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{i,t-1}$$
(A.2)

$$\tau_t = m^2 + \theta^2 \sum_{k=1}^{K_v} \varphi_k(\omega_v) R V_{t-k}$$
(A.3)

In the long-run component, the realized variance RV_t of quarter *t* is the sum of squared daily returns in the quarter. And the weight function φ_k follows the beta polynomials with the decay parameter ω_v . The larger the ω_v is, higher weight are attached to the most recent quarters. The smaller the ω_v is, the smoother the weights are across the horizon.

$$RV_t = \sum_{i=1}^{N_t} r_{i,t}^2$$
(A.4)

$$\varphi_k(\omega_v) \propto (1 - \frac{k}{K_v})^{\omega_v - 1} \tag{A.5}$$

A.2 DCC model

Colacito, Engle, and Ghysels (2011) further extend the GARCH-MIDAS model to accommodate the multivariate case with dynamic correlations. In our study regarding the correlation between stock and bond, the DCC-MIDAS model decomposes the 2×2 conditional covariance matrix H_t into the diagonal conditional variance matrix D_t and the conditional correlation matrix R_t . D_t matrix has conditional stock/bond volatility $\sqrt{\tau_t g_{i,t}}$ as diagonal element, specified in the first step. While the R_t is a rescale of the quasi-correlation matrix Q_t so that the diagonals are unity.

$$H_t = D_t R_t D_t \tag{A.6}$$

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2}$$
(A.7)

Assume the quasi-correlation matrix Q_t whose diagonal elements $q_{s,b,i,t}$ and off-diagonal elements $q_{s,s,i,t}$, $q_{b,b,i,t}$ have the GARCH(1,1)-like dynamics. For instance, the short run stock-bond correlation $q_{s,b,i}$ has the following dynamics, where $\bar{\rho}_{s,b,t}$ is the long-run stock-bond correlation component and $\xi_{i-1,t}$ is the standardized residuals computed in the first step

$$q_{s,b,i,t} = \bar{\rho}_{s,b,t} (1 - a - b) + a\xi_{s,i-1,t} \xi_{b,i-1,t} + bq_{s,b,i-1,t}$$
(A.8)

Similar to Equation(A.3), the long-run correlation component $\bar{\rho}_{s,b,t}$ is the weighted sum of K_c lags of realized correlation $c_{s,b,t}$, calculated on N_t non-overlapping standardized residuals. And the weight function φ_k follows the beta polynomials with the decay parameter ω_c .

$$\bar{\rho}_{s,b,t} = \sum_{l=1}^{K_c} \varphi_k(\omega_c) c_{s,b,t-1} \tag{A.9}$$

$$c_{s,b,t} = \frac{\sum_{i=1}^{N_t} \xi_{s,i} \xi_{b,i}}{\sqrt{\sum_{i=1}^{N_t} \xi_{s,i}^2 \sum_{i=1}^{N_t} \xi_{b,i}^2}}$$
(A.10)

$$\varphi_k(\omega_c) \propto (1 - \frac{k}{K_c})^{\omega_c - 1} \tag{A.11}$$

A.3 Estimation

TableA.1 displays the estimation results for the DCC-MIDAS model using the likelihood profiling procedure. When estimating the stock-Treasury and stock-HY correlation, we set lags K_v to 6 and K_c to 12 following (Colacito, Engle, and Ghysels, 2011). We report all parameters involving twostep estimation procedure, including conditional stock variance, conditional bond variance and the conditional stock-bond correlation. It can be noticed that most parameters are significant at the 1% level.

Table A.1 Estimate for DCC-MIDAS model

This table reports the parameter estimates for DCC-MIDAS model. Panel A reports the results for stock-Treasury correlation. Panel B presents the estimates for stock-HY correlation. Standard errors are in parentheses. *, **, and *** denotes significance at the 10%, 5%, and 1% level, respectively

Panel A: Stock-T	reasury Corre	elation				
	α	β	$\mu(imes 10^3)$	$m(\times 10^2)$	θ	ω_c
Stock Variance	0.112***	0.856***	0.558***	0.577***	0.099***	3.120***
	(0.003)	(0.005)	(0.059)	(0.027)	(0.004)	(0.429)
Bond Variance	0.090***	0.879***	0.044***	0.019	0.140***	2.119***
	(0.003)	(0.005)	(0.017)	(0.014)	(0.003)	(0.174)
	а	b	ω_c			
Correlation	0.050***	0.905***	3.353***			
	(0.003)	(0.007)	(0.439)			
Panel B: Stock-H	IY Bond Corr	relation				
	α	β	$\mu(imes 10^3)$	$m(\times 10^2)$	θ	ω _c
Stock Variance	0.116***	0.852***	0.681***	0.697***	0.089***	2.052***
	(0.006)	(0.008)	(0.092)	(0.042)	(0.006)	(0.456)
Bond Variance	0.393***	0.600***	0.471***	0.671***	0.152***	5.110***
	(0.009)	(0.007)	(0.013)	(0.188)	(0.043)	(1.444)
	а	b	ω_c			
Correlation	0.000	0.000	4.194***			
	(0.007)	(1.346)	(0.594)			

B Stock-IG Correlation

We extract the realized and MIDAS correlation between stock and IG corporate bond excess return via DCC-MIDAS model. Figure B.1 reveals the striking similarity in the patterns of stock-IG correlation and stock-Treasury correlation. The correlation between stock and IG corporate bond is positive until the year 1998, after which it begins to turn negative. In fact, the correlation between these two indicators is as high as 0.84 and they share the same break point 1998 Q1. IG corporate bonds resemble U.S. Treasury bonds in several ways due to their relatively low credit risk compared to lower-rated corporate bonds. Both Treasury and IG bond possess high credit quality, low yields, high market liquidity and share the underlying interest rate risk, which shapes the nearly identical comovements between them.





This figure plots the realized quarterly correlation and MIDAS correlation between ten-year Treasury and ICE investment grade corporate bond index returns following Colacito, Engle, and Ghysels (2011). Sample period is from 1991 Q1 to 2020 Q4.